Renormalisation-Group Induced Moduli Stabilisation and Inflation

Fernando Quevedo
University of Cambridge

String Phenomenology 2022

Liverpool July 2022

C.P. Burgess + FQ: 2202.05344 (JHEP) + …

In memoriam

Graham Ross (1944-2021)

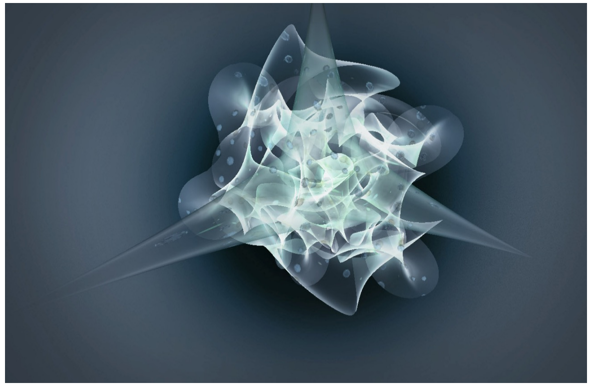
Costas Kounnas (1952-2022)

Outline

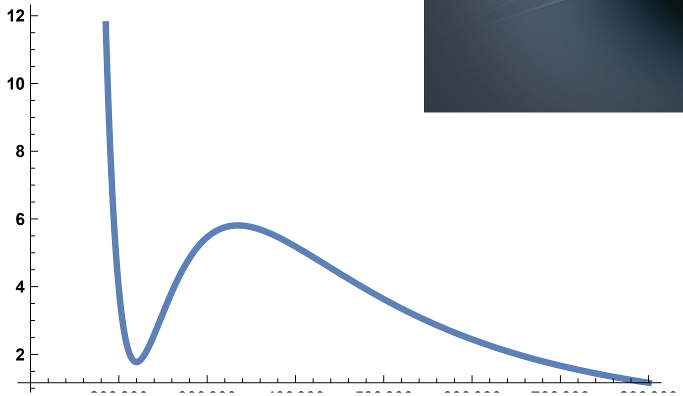
- Review flux compactifications
- RG-induced moduli stabilization
- Brane anti-brane inflation revamped

Three related questions

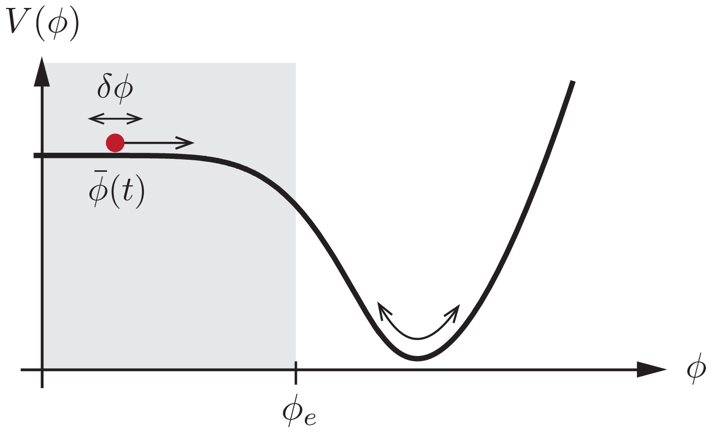
- Moduli stabilization



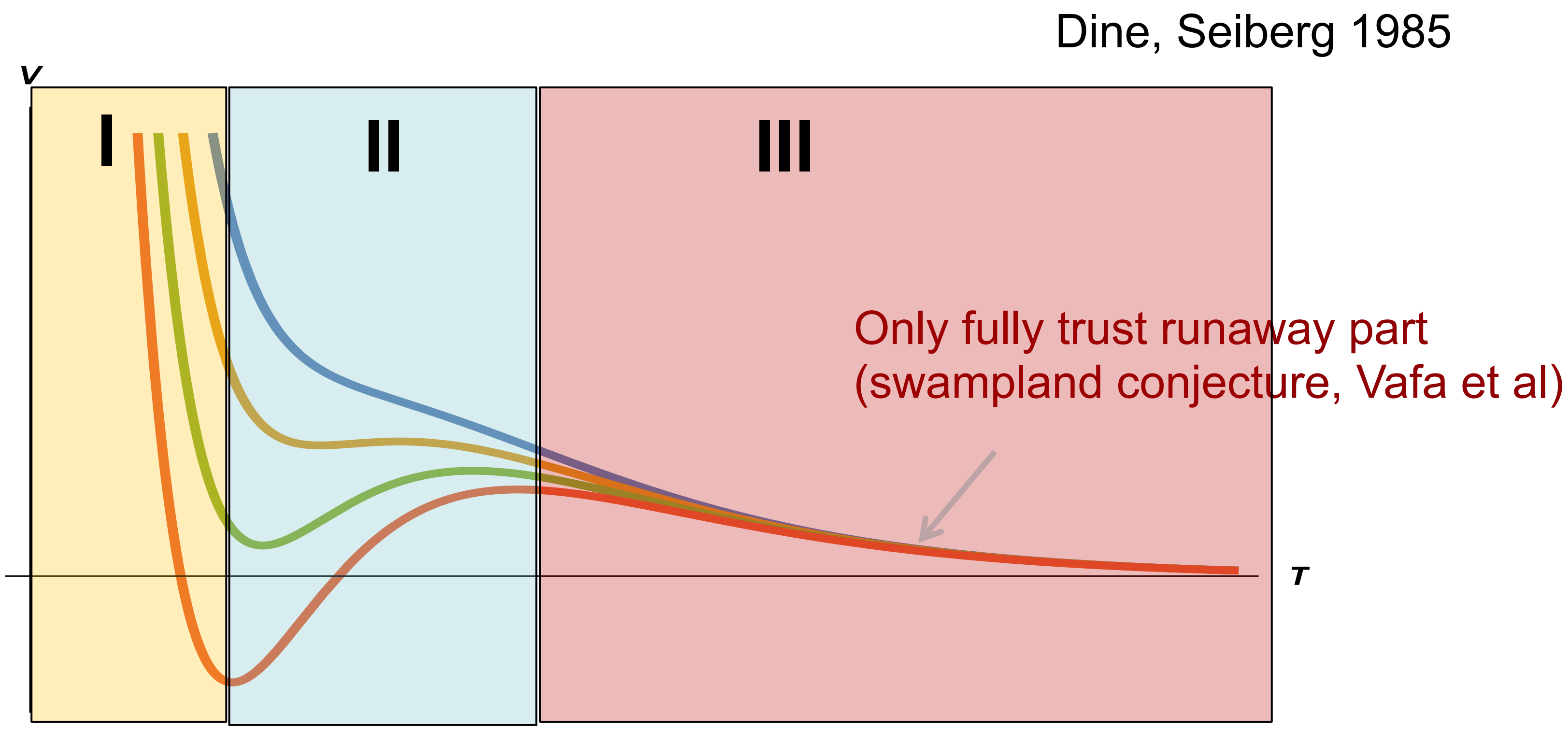
- De Sitter



- Inflation



Dine-Seiberg Problem



V $\longrightarrow$ **0** at weak coupling and large volume.

e.g.Loop and α' corrections in IIB

$$S_{\rm bulk} = \int {\rm d}^{10}x\,\sqrt{-\tilde g}\,\left\{\tilde R - \frac{|\partial \mathcal{S}|^2}{({\rm Re}\,\mathcal{S})^2} - \frac{|G_{(3)}|^2}{{\rm Re}\,\mathcal{S}} - \tilde F_{(5)}^2\right\} + \int \frac{1}{{\rm Re}\,S}\,C_{(4)}\wedge G_{(3)}\wedge \overline{G}_{(3)}$$

Scalings

$$\tilde g_{MN} \to \tilde g_{MN}\,, \qquad \mathcal{S} \to a^2 \mathcal{S}\,, \qquad G_{(3)} \to a G_{(3)}\,, \qquad \tilde F_{(5)} \to \tilde F_{(5)}$$

$$S \to \frac{a\,\mathcal{S} - ib}{ic\,\mathcal{S} + d} \quad \text{and} \quad G_{(3)} \to \frac{G_{(3)}}{ic\,\mathcal{S} + d}$$

$$\tilde g_{MN} \to \lambda \tilde g_{MN}\,, \qquad \mathcal{S} \to \mathcal{S}\,, \qquad B_{(2)} \to \lambda B_{(2)}\,, \qquad C_{(2)} \to \lambda C_{(2)}\,. \qquad C_{(4)} \to \lambda^2 C_{(4)}\,.$$

$$e^{-K/3} = s^{1/3}\tau \sum_{nmr} \mathcal{A}_{nmr} \left(\frac{1}{s}\right)^n \left(\frac{s}{\tau}\right)^{(m+r)/2}$$

$$s = {\rm Re}\,\mathcal{S} = e^{-\phi}$$

$$\mathcal{V} \propto \tau^{3/2}$$

$$T = \tfrac{1}{2}(\tau + i\mathfrak{a})$$

$$V(s,\tau) = \sum_{nm} A_{nm} s^{-n}\tau^{-m}$$

Burgess, Cicoli, Krippendorf, FQ 2020
Cicoli, FQ, Savelli, Schachner, Valandro 2021

Dine-Seiberg:
If it has a minimum is generically at s, τ of order 1

Quick Overview of Flux compactifications

- **Tree-level Kahler potential:**

$$K_{tree} = -2\ln \mathcal{V}(T_i + \overline{T}_i) - \ln(S + \overline{S}) - \ln\left(i \int_{CY} \Omega(U) \wedge \overline{\Omega} \right)$$

- **Tree-level superpotential**:

$$W_{tree} = \int_{CY} G_3 \wedge \Omega(U) \qquad G_3 = F_3 + iSH_3$$

- **Flux quantisation:**

$$\frac{1}{2\pi\alpha'} \int_{\Sigma_3^k} H_3 = n_k \qquad \frac{1}{2\pi\alpha'} \int_{\Sigma_3^k} F_3 = m_k \qquad \mathrm{k} = 1,\ldots, n = 2h^{1,2} + 2$$

→ **free parameters** (n_k, m_k)

$$D_S W = 0 \qquad D_{U_\alpha} W = 0 \quad \Rightarrow \quad W_0 \equiv \langle W_{tree} \rangle$$

$$DW \equiv \partial W + W \partial K$$

Can fix all U and S ($\mathrm{S} \sim m/n \gg 1$)

$$N_{sol} \approx 10^{2n} = 10^{4(h^{1,2}+1)} \approx 10^{400} \qquad \text{for } h^{1,2} \approx O(100)$$

String Landscape

Flux Compactifications in IIB String Theory

- **S,U,T Moduli**

$$V_F = e^K \left(K^{-1}_{M\overline{N}} D_M W \overline{D}_{\overline{M}} \overline{W} - 3|W|^2 \right)$$

$$W_{\text{tree}} = W_{\text{flux}}(U, S) \qquad K^{-1}_{i\bar{j}} K_i K_{\bar{j}} = 3$$

No-scale

$$V_F = e^K \left(K^{-1}_{a\bar{b}} D_a W D_{\bar{b}} W \right) \geq 0$$

Fluxes fix S,U but T flat

- **Quantum corrections** $\delta V \propto W_0^2 \delta K + W_0 \delta W$
- **Three options:**

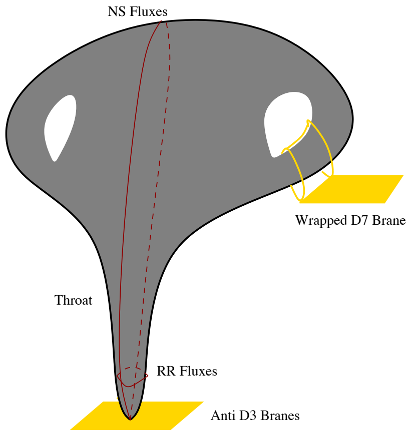
(i) $W_0 \gg \delta W \quad \delta K \gg \delta W$ **Perturbative Runaway: Dine-Seiberg problem…?**

(ii) $W_0 \sim \delta W = W_{\text{np}}.$

$W_0 \ll 1$ **Fix T-modulus: KKLT**

(iii) $\delta K \sim W_0 \delta W$

$\delta K \sim 1/\mathcal{V}$ and $\delta W \sim e^{-a\tau}$ **Fix T-moduli: LVS**



Cascales et al
hep-th/0312051

Challenges to KKLT, LVS,...

- **Fluxes under control only in SUSY 10D?** (Sethi, Kachru-Trivedi, de Alwis et al…)
- **All SUSY breaking part is 4D EFT. Trust EFT?**(Carta, et al, Moritz et al, Kallosh, Gautason et al, Hamada et al, Kachru et al.)
- **Higher corrections in LVS?** (Cicoli et al.)
- **Antibranes (non susy, singularity?)** (Bena et al, Moritz et al, Cohen-Maldonado et al, Gao et al)
- **Tadpole problem** (Bena et al., Crino et al, Junghans, Xin Gao et al, Vafa et al…)
- **Consistency with AdS/CFT** (de Alwis et al, Conlon et al, Vafa et al…)
- **Tuning W_0<<1? in KKLT** (Demirtas et al, Alvarez-Garcia et al, Blumenhagen et al)

RG induced moduli stabilization
An Alternative to KKLT/LVS?

C.P. Burgess + FQ 2202.05344

Logarithmic Corrections and the RG

$$K(T,\overline{T}) = -3\ln\mathcal{P}, \quad \text{with} \quad \mathcal{P}(\tau) = \tau\left[1 - \frac{k}{\tau} + \frac{h}{\tau^{3/2}} + \mathcal{O}\left(\frac{1}{\tau^2}\right)\right]$$

But in principle there should be logarithmic dependence at each order

$$\mathcal{A}_{nmr} = \mathcal{A}_{nmr}(\log\tau)$$

Albrecht et al 2001, Aghababaie et al 2002
Conlon et al 2010, Grimm et al 2015
Weissenbacher 2019, Weigand et al 2019,
Antoniadis et al 2019

$$V = \frac{3(k' - k'')}{\tau^4}|w_0|^2 + \mathcal{O}(\tau^{-9/2})$$

k constant: extended no-scale

$$k \simeq k_0 + k_1\,\alpha_g + \frac{k_2}{2}\,\alpha_g^2 + \cdots$$

$$\tau\,\frac{\mathrm{d}\alpha_g}{\mathrm{d}\tau} = \beta(\alpha_g) = b_1\alpha_g^2 + b_2\,\alpha_g^3 + \cdots.$$

$$\alpha_g(\tau) = \frac{\alpha_{g0}}{1 - b_1\,\alpha_{g0}\,\ln\tau},$$

$$V(\tau) \simeq \frac{U(\ln\tau)}{\tau^4}$$

$$U \simeq U_1\,\alpha_g^2 - U_2\,\alpha_g^3 + U_3\,\alpha_g^4 + \cdots,$$

$$\alpha_{g0}\ln\tau_0 \simeq \mathcal{O}(1)$$

Dine-Seiberg argument implies exponentially large τ !

Concrete Example:

$$k := \mathcal{K}\left(\log\tau\right)$$

$$V_F \simeq \frac{\mathcal{C}}{\tau^4}\left(\mathcal{K}' - \mathcal{K}''\right) + \mathcal{O}(\tau^{-9/2})\,, \qquad \mathcal{C} := 3|w_0|^2$$

$$\mathcal{K} \simeq \mathcal{K}_0 + \mathcal{K}_1\left(\frac{\alpha_g}{4\pi}\right) + \frac{\mathcal{K}_2}{2}\left(\frac{\alpha_g}{4\pi}\right)^2 + \cdots$$

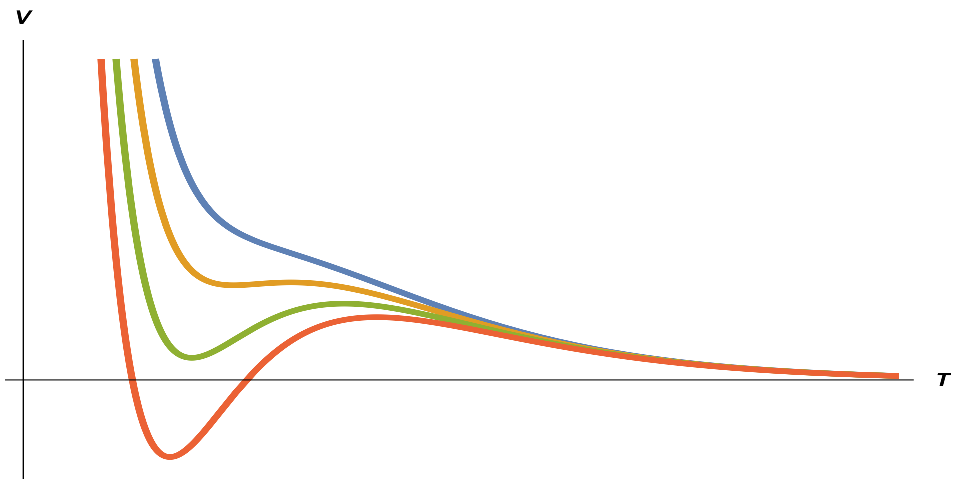
$$\frac{4\pi}{\alpha_g} = b_0 - b_1 \ln\tau$$

$$V_F \simeq \frac{\mathcal{C}}{\tau^4}\Big[\mathcal{K}_1 b_1\left(\frac{\alpha_g}{4\pi}\right)^2 + \left(\mathcal{K}_1 b_2 + \mathcal{K}_2 b_1 - 2\mathcal{K}_1 b_1^2\right)\left(\frac{\alpha_g}{4\pi}\right)^3 + \left(\mathcal{K}_1 b_3 + \mathcal{K}_2 b_2 + \mathcal{K}_3 b_1 - 5\mathcal{K}_1 b_1 b_2 - 3\mathcal{K}_2 b_1^2\right)\left(\frac{\alpha_g}{4\pi}\right)^4 + \cdots\Big].$$

if $|\mathcal{K}_2/\mathcal{K}_3| \sim \mathcal{O}(\epsilon)$ and $|\mathcal{K}_1/\mathcal{K}_3| \sim \mathcal{O}(\epsilon^2)$

$$\alpha_{0\pm} \simeq \frac{1}{2}\left[-\frac{\mathcal{K}_2}{\mathcal{K}_3} \pm \sqrt{\left(\frac{\mathcal{K}_2}{\mathcal{K}_3}\right)^2 - \frac{4\mathcal{K}_1}{\mathcal{K}_3}}\right] \sim \mathcal{O}(\epsilon)$$

$$\ln\tau_0 \sim 1/\alpha_0 \sim \mathcal{O}(1/\epsilon)$$



AdS and dS minima

$$\tau \simeq e^{1/\epsilon} \gg 1$$

SUSY Breaking

$$F^T = e^{K/2} K^{T\overline{T}} K_T W \sim \frac{w_0}{\tau^{1/2}} + \mathcal{O}(\tau^{-3/2})$$

$$m_\tau = \left(\frac{\tau^2}{M_p^2}\frac{\partial^2 V}{\partial \tau^2}\right)^{1/2} \sim \frac{\epsilon^{5/2}|w_0|}{\tau^2 M_p^2} \sim \frac{\epsilon^{5/2} m_{3/2}}{\tau^{1/2}} \quad \text{where} \quad m_{3/2} \sim \frac{|w_0|}{\tau^{3/2} M_p^2}$$

To avoid cosmological moduli problem $\quad m_\tau \sim 30 \text{ TeV}$

$$\tau_0 \sim 10^6\,, \quad m_{3/2} \sim 10^9 \text{ GeV}\,, \quad M_{KK} \sim 10^{12} \text{ GeV}\,, \quad M_s \sim 10^{14} \text{ GeV} \quad \text{if} \quad |w_0| \sim M_p^3$$

Soft terms

$$m_\psi^2 = m_{3/2}^2 - F^i F^{\bar{j}} \partial_i \partial_{\bar{j}} \ln Z_\psi \quad \text{which implies} \quad m_\psi \sim \frac{w_0}{\tau^2} \sim \frac{m_{3/2}}{\tau^{1/2}}$$

$$M_G = \frac{F^i \partial_i f}{\text{Re} f} \sim \frac{w_0}{\tau^{5/2}} \sim \frac{m_{3/2}}{\tau}$$

Split SUSY

Revisiting Brane Anti-brane Inflation

Recall Brane-Antibrane Inflation

- Interactions calculable

$$V(\phi) = 2T_3 \left(1 - \frac{1}{2\pi^3} \frac{T_3^3}{M_{10,Pl}^8 \phi^4}\right)$$

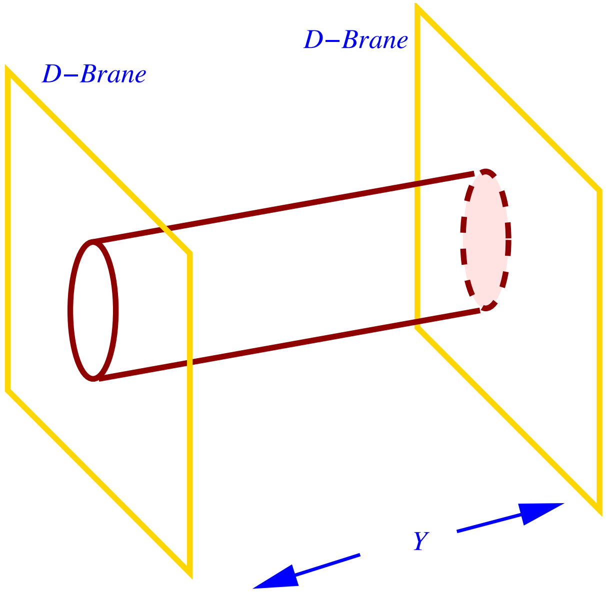
- End by tachyon condensation

Burgess et al 2001

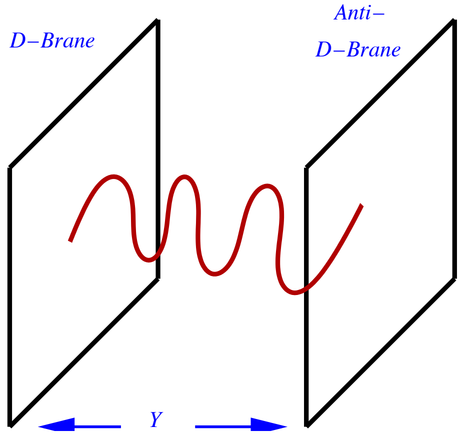
- But no slow roll

$$\eta = -\frac{10}{\pi^3}(L/r)^6$$

- No moduli stabilisation



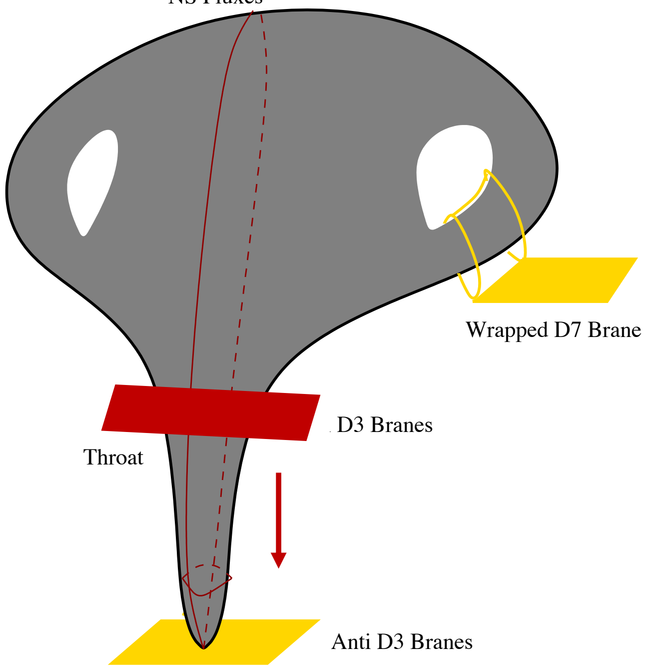
Dvali and Tye 1998



Burgess et al 2001
Dvali et al. 2001

Recall: Warped D3-$\overline{\text{D3}}$ Inflation

KKLMMT 2003



$$ds^2 = \left(1 + \frac{e^{-4\mathcal{A}}}{\mathcal{V}^{2/3}}\right)^{-1/2} ds_4^2 + \left(1 + \frac{e^{-4\mathcal{A}}}{\mathcal{V}^{2/3}}\right)^{1/2} ds_{CY}^2$$

$$e^{4\mathcal{A}_0} := e^{-4\rho} = e^{-\frac{8\pi K}{3 g_s M}}$$

$$V = 2T_3 e^{-4\rho}\left(1 - \frac{27}{64\pi^2}\frac{2T_3 e^{-4\rho}}{\phi^4}\right) := \Omega\left(1 - \frac{\gamma\Omega}{\phi^4}\right)$$

$$\Omega = \frac{\alpha e^{-4\rho}}{\mathcal{V}^{4/3}} M_p^4,$$

$$\epsilon := \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2 \simeq 8\gamma^2\left(\frac{\Omega M_p}{\phi^5}\right)^2$$

$$\eta := M_p^2 \frac{V_{\phi\phi}}{V} \simeq -20\gamma\frac{\Omega M_p^2}{\phi^6}$$

$$\eta = -\frac{5}{6N_e}, \qquad \epsilon = \frac{20\pi}{9\sqrt{2}}\frac{\delta_H}{N_e^{5/2}}$$

$$\epsilon \ll 1 \text{ and } \eta \ll 1$$

Eta problem

$$W = W(T, \Phi)$$

$$\mathcal{V} = \left(\tau - \bar{\phi}\phi\right)^{3/2}$$

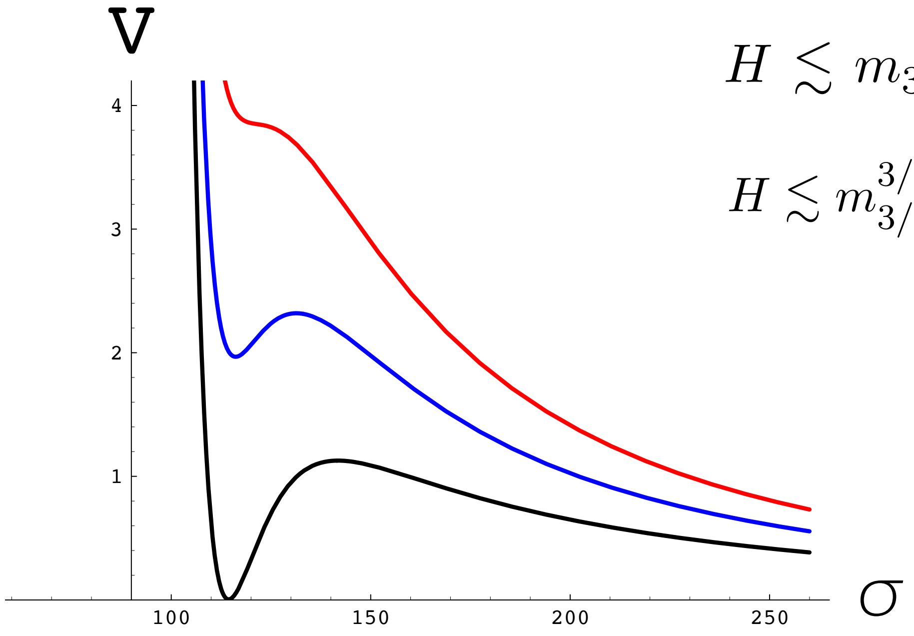
$$V = e^K\, \widehat{V}_0 \simeq \frac{\widehat{V}_0}{[\tau - \bar{\phi}\,\phi + \cdots]^3} \simeq \frac{\widehat{V}_0}{\tau^3}\left[1 + \frac{3\bar{\phi}\,\phi}{\tau} + \cdots\right] \simeq \frac{\widehat{V}_0}{\tau^3}\Big[1 + \bar{\varphi}\,\varphi + \cdots\Big]\,.$$

eta problem!

Challenge: Find a W(Φ) that can implement the fine tuning: inflection point inflation

Baumann et al 2007-2009

Also: Kallosh-Linde problem

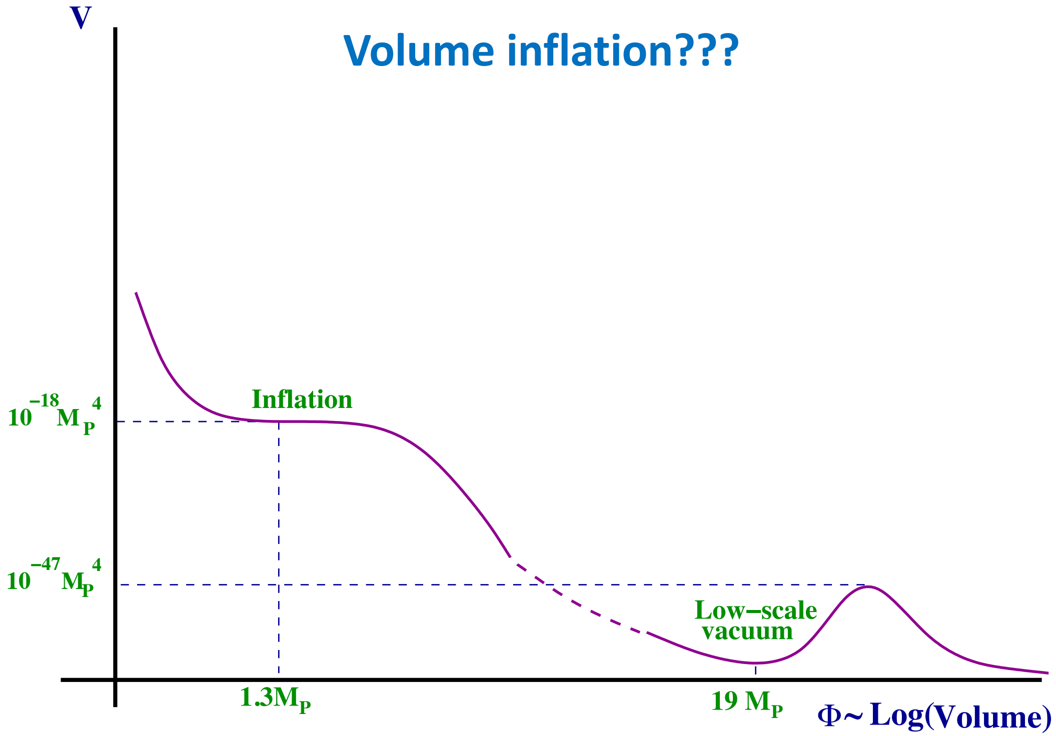


Kallosh, Linde 2004

$H \lesssim m_{3/2}$ KKLT

$H \lesssim m_{3/2}^{3/2}$ LVS

Volume inflation???



Conlon, Kallosh, Linde, FQ 2008

Non-linear SUSY and Inflation

Recall: Nonlinear SUSY and KKLT

Goldstino superfield

$$X^2(x,\theta) = 0.$$

Rocek,...,Komargodski, Seiberg,...

$$X = X_0(y) + \sqrt{2}\psi(y)\theta + F(y)\theta\bar{\theta} \qquad X_0 = \frac{\psi\psi}{2F}$$

KKLT

$$K = -3\log\left(T+T^*\right) + c\left(T+T^*\right)^n \; XX^* + ZCC^* + \cdots$$

$$W = W_0 + W_{\rm matter} + W_{np} + \boxed{\rho X}$$

Plug into SUGRA expression for V, V= V_{KKLT} + V_{uplift} :

$$V_{\rm uplift} = \frac{|\rho|^2}{c(T+T^*)^{n+3}}$$

(just like KKLT, KKLMMT!)

Antibrane uplift from manifestly SUSY EFT!

Ferrara, Kallosh, Linde,… 2013-15
Polchinski @ SUSY 2015

Non-Linear SUSY and Inflation

- Supersymmetric gravity but SM broken SUSY (non-linearly realised)

Goldstino superfield: **X** $$X^2 = 0$$

- Inflation mechanism to also reduce the leading contribution to *Λ*

Inflaton (relaxon) superfield: **Φ** $$\overline{\mathcal{D}}(X\overline{\Phi}) = 0 \qquad X\overline{\Phi} = X\Phi$$

- Accidental approximate scale invariance

Dilaton superfield: **T** $$\mathcal{T} = \frac{1}{2}(\tau + i\mathfrak{a})$$

Also C.P. Burgess, D. Dineen, FQ 2021

Reconsider Brane-Antibrane Inflation

- RG-induced moduli stabilization

- Non-linear supersymmetry

Low Energy Effective Action

$$K(T,\overline{T},X,\overline{X},\Phi,\overline{\Phi}) \simeq -3M_p^2 \ln \mathcal{P}$$

$$\mathcal{P}(\tau,X,\overline{X},\Phi,\overline{\Phi}) = \tau - k + \frac{h}{\tau} + \cdots$$

$\overline{\mathcal{D}}(X\overline{\Phi}) = 0$

$X(\Phi - \overline{\Phi}) = 0.$

$$W \simeq w_0(\Phi) + X w_X(\Phi,\overline{\Phi})$$

$$k = \frac{1}{M_p^2}\Big[\mathfrak{K}(\Phi,\overline{\Phi},\ln\tau) + (X+\overline{X})\mathfrak{K}_X(\Phi,\overline{\Phi},\ln\tau) + \overline{X}X\mathfrak{K}_{X\overline{X}}(\Phi,\overline{\Phi},\ln\tau)\Big],$$

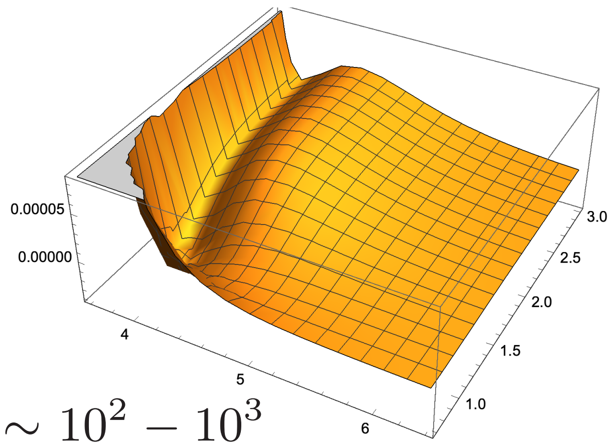
$$V_F \simeq \underbrace{\frac{1}{\mathcal{P}^2}\left[\frac{1}{3}\right.}_{\mathcal{O}(\tau^{-2})}\mathfrak{K}^{\overline{X}X}\overline{w_X}w_X + \frac{\mathfrak{K}^{\overline{X}X}\mathfrak{K}_{X\overline{T}}}{M_p^2}\underbrace{w_0\overline{w_X} + \frac{\mathfrak{K}^{\overline{X}X}\mathfrak{K}_{T\overline{X}}}{M_p^2}}_{\mathcal{O}(\tau^{-1})}w_X\overline{w_0} - \frac{3(\mathfrak{K}_{T\overline{T}} - \mathfrak{K}^{\overline{X}X}\mathfrak{K}_{T\overline{X}}\mathfrak{K}_{X\overline{T}})}{1+\underbrace{2\mathfrak{K}^{X\overline{X}}\mathfrak{K}_X\mathfrak{K}_{\overline{X}}/M_p^2}_{\mathcal{O}(\tau^{-2})}}\left.\frac{|w_0|^2}{M_p^4}\right]$$

$$w_X \simeq 0$$

Inflationary potential

$$V \sim (A w_X^2 \tau^2 - B w_X \tau + C)/\tau^4,$$

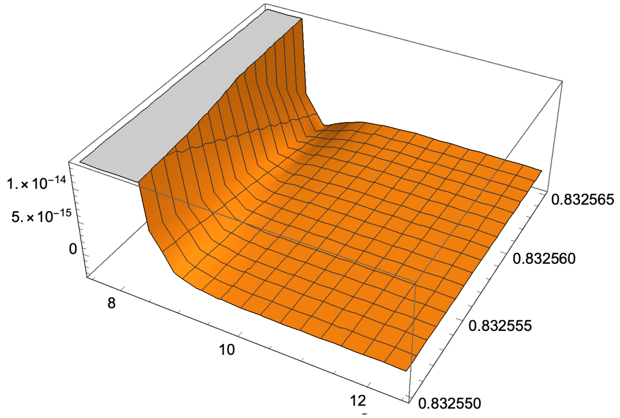
$$W = W_0 + X w_X, \qquad w_X = A - \frac{B}{\Phi^4}$$



$\tau \sim 10^2 - 10^3$

$w_X \neq 0$

High scale inflationary region



$\tau \sim 10^{26}$

$w_X \sim 0,$

Low scale late time minimum

Warped D3-$\overline{\text{D3}}$ Inflation Reconsidered

- **If moduli stabilization is perturbative: No eta problem!**
- **Coulomb potential from SUSY Nilpotent formalism**

$$W = W_0 + X w_X, \qquad w_X = A - \frac{B}{\Phi^4}$$

Brane separation as 'relaxon'!

- **Gravitino mass during inflation>>than after (no KL problem!)**
- **Slow-roll**

$$\varepsilon = \frac{M_p^2}{2}\left(\frac{V_\varphi}{V}\right)^2 \simeq 8\mathfrak{b}^2\left(\frac{\Omega M_p}{|\varphi|^5}\right)^2 \quad \text{and} \quad \eta = \frac{M_p^2 V_{\varphi\varphi}}{V} \simeq -\frac{20\mathfrak{b}\,\Omega M_p^2}{|\varphi|^6} \qquad \varepsilon \ll |\eta| \ll 1$$

$$r = 16\varepsilon_* \simeq \frac{64\pi}{5}\sqrt{\frac{3}{10}}\,\delta_H |n_s - 1|^{5/2} \simeq 2 \times 10^{-8}$$

$$N_e = \frac{1}{M_p}\int_{\varphi_{end}}^{\varphi_*} \frac{d\varphi}{\sqrt{2\varepsilon}} \simeq \frac{\varphi_*^6}{24\mathfrak{b}\,\Omega M_p^2} \simeq 56$$

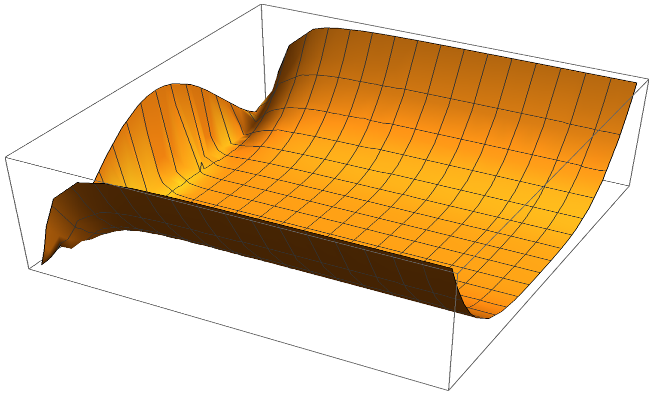
Ending inflation

- **Higgs field as the tachyon to end inflation?**

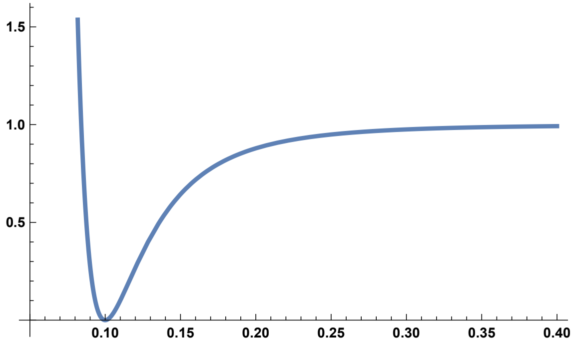
$$W = w_0 + Xw_X \quad \text{with} \quad w_X = \mathfrak{t} - \frac{\mathfrak{g}}{|\Phi|^4} - \lambda|\mathcal{H}|^2$$

$$V \propto \frac{|w_X|^2}{\mathcal{P}^2} = \frac{\left(\mathfrak{t}|\phi|^4 - \mathfrak{g}\right)^2}{\mathcal{P}^2|\phi|^8} + \frac{2\lambda\left(\mathfrak{t}|\phi|^4 - \mathfrak{g}\right)}{\mathcal{P}^2|\phi|^4}|\mathcal{H}|^2 + \frac{\lambda^2|\mathcal{H}|^4}{\mathcal{P}^2}$$

$$m^2 = \frac{2\lambda}{\mathcal{P}}\left(\mathfrak{t} - \frac{\mathfrak{g}}{|\phi|^4}\right) < 0$$



- **Supersymmetric Coulomb potential ?**



Also: Aparicio, FQ, Valandro 2015

Conclusions

- Renormalisation group may address Dine-Seiberg problem
- Perturbative moduli stabilization
- Supersymmetric treatment of brane-antibrane inflation (no eta nor KL problems!)
- Many open questions (end of inflation, explicit string models?)